DYNAMICS OF THE CAVITATION ZONE DURING AN UNDERWATER EXPLOSION NEAR A FREE SURFACE

V. K. Kedrinskii

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The problem of the development of the cavitation zone and the rarefaction wave profile in the region of regular reflection of the spherical shock wave of an underwater explosion from a free surface is analyzed for the axisymmetric formulation within the framework of a model of the two-phase medium consisting of a liquid with cavitation nuclei of the free gas uniformly distributed in it. An example of the calculation of the rarefaction wave profile and the zone of visible cavitation at different times is given for the case of the explosion of 1-g charge at depths of 3 and 5.3 cm for an initial volumetric gas concentration of 10^{-11} and an initial cavitation nucleus radius of $5 \cdot 10^{-5}$ cm. The results of the calculation are compared with experiment.

INTRODUCTION

The question of the development of cavitation and the structural properties of the wave pattern near the free surface of a liquid when the shock wave of an underwater explosion is reflected from it has been analyzed in [1-6].

The parameters of the shock wave have been studied experimentally and an analysis has been made of the development of the cavitation zone during the explosion of charges with weights of 1 g and 100 kg at depths of 1-8 relative to the radii of the charges [1]. The zones of negative pressures near the free surface during the underwater explosion of charges of 50, 100, and 5000 g at depths of 1-12 m have been calculated on a computer in the acoustical approximation [4]. The zone of nonlinear interaction of the shock wave of an underwater explosion with the free surface has been determined [2, 3]. The zone is bounded below by the trajectory of the "triple" point at which the front of the attenuated shock wave, the undisturbed front of the incident wave, and the rarefaction wave front converge. The "smooth" reduction in pressure behind the front of the disturbance wave, which is characteristic of a nonlinear zone, gives reason to assume that visible cavitation discontinuities do not develop in this zone. Consequently, the development of cavitation can be observed only in the region of regular reflection, for which the acoustical approximation [2, 6], using the principle of superposition of the pressure field from the explosion of an imaginary charge, is valid. Within the framework of a one-phase liquid, however, the use of this principle leads to overstatement (sometimes by several orders of magnitude) of the absolute values of the negative pressures compared with their true values [6]. In this connection the results of the study of the strength characteristics of liquids [7] are used in some works [5, 6], assuming that liquids do not withstand large tensions, the limiting value of which should have the order of the vapor saturation pressure, i.e., should be close to zero.

It is known [8] that real liquids, including distilled water, contain free gas for which the volumetric concentration and the size of the cavitation nuclei are determined by the state of the liquid. In connection with this it is quite natural in models of the development of the cavitation zone to take into account the already existing gas bubbles and to determine their effect on the process under investigation, without addressing the problem of the formation of the cavitation nuclei. In this sense the closest to this problem are [9, 10], in which the conditions of the start of the growth of a single spherical gas bubble in a viscous incompressible liquid under the effect of a sharp pressure drop are studied. According to [10], the cavitation

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is characterized primarily by the unbounded bubble growth (at a constant negative pressure), and in this case the decisive role is played by the amount of gas in the bubbles and not by the viscosity and the thermodynamic state. This formulation is interesting and in combination with the acoustical method of determining the negative pressure it can be used for certain estimates of the development of the cavitation zone.

An approach like that of [9, 10] cannot describe an actual process, since it does not allow for the interaction of the gas cavities during their expansion and the effect of their dynamics on the tensile stresses in the cavitation zone. In connection with this, the principal attention in the present work will be paid to a new formulation: the development of cavitation near the free surface of a liquid containing bubbles of free gas upon the application of negative pressure. The approach to the problem is similar to that of [11] in many ways.

1. Estimate of the Cavitation Region Based on the Dynamics of a Single Gas Bubble

Following [9, 10], let us consider the problem of the behavior of a cavitation nucleus in a rarefaction wave in the case of the instantaneous application of negative pressure (the liquid is ideal and incompressible).

We introduce the dimensionless variables and parameters

$$R = R_0 y;\quad t = R_0 \sqrt{\rho_0/p_0}\,\tau;\quad p = p_0 P;\quad W = \frac{2\sigma}{R_0 p_0},$$

where R_0 and p_0 are the initial radius of the bubble and gas pressure in it; σ is the surface-tension coefficient; ρ_0 is the density of the liquid. The equation of pulsation of the bubble has the form

$$y\ddot{y} + 3/2\dot{y}^2 = y^{-3\gamma} - \frac{W}{y} - P. \tag{1.1}$$

When $\tau = 0$, $y = 1$, and $\dot{y} = 0$

$$P = \begin{cases} -\pi^0 & 0 \leqslant \tau \leqslant \sqrt{p_0/\rho_0}\,T/R_0, \\ p_\infty/p_0 & \tau > \sqrt{p_0/\rho_0}\,T/R_0, \end{cases}$$

where p_∞ is the undisturbed pressure at infinity. The value of p_0 is determined from the condition of equilibrium of the bubble when $\tau < 0$ ($p = p_\infty$):

$$p_0 = p_\infty + 2\sigma/R_0.$$

Equation (1.1) can be represented in the form

$$\frac{d}{dy}\, y^3\dot{y}^2 = 2y^{2-3\gamma} - 2yW - 2y^2P. \tag{1.2}$$

The first integral of Eq. (1.2) with $P = -\pi^0 = \text{const}$ has the form

$$\dot{y}^2 = \frac{2}{3}\, y^{-3}\left[\left(y^{3-3\gamma} - 1\right) - \frac{3}{2} W\left(y^2 - 1\right) + \pi^0\left(y^3 - 1\right)\right].$$

It can be noted that $\dot{y}$ rapidly reaches the asymptotic value $(^2/_3\pi^0)^{1/2}$ when $\pi^0 \gg 1$. In this case the current value of the radius of the cavitation nucleus is determined with a sufficient degree of accuracy by the expression

$$y \simeq \left(\frac{2}{3}\,\pi^0\right)^{1/2} \tau. \tag{1.3}$$

If at $t = T$ the pressure again becomes equal to atmospheric pressure (p_∞), then the next stage of the expansion of the bubble to its maximum size is determined by the inertial motion for $t \geq T$:

$$\frac{d}{dy}(y^3\dot{y}^2) \simeq -2y^2 p_\infty/p_0$$

with the initial values

$$\dot{y}_1 = \left(\frac{2}{3}\pi^0\right)^{1/2}, \quad y_1 = \left(\frac{2}{3}\pi^0 p_0/\rho_0\right)^{1/2} T/R_0,$$

from which the maximum bubble radius is determined by the expression

$$y_{\max} = y_1\left(1 + \frac{3}{2}\dot{y}_1^2 p_0/p_\infty\right)^{1/3} \tag{1.4}$$

or

$$y_{\max} \simeq 0.815(\pi^0 p_0)^{5/6} T R_0^{-1} \rho_0^{-1/2} p_\infty^{-1/3}.$$

The expressions (1.4) or (1.3) allow one to determine the maximum radius of a cavitation bubble expanding in a rarefaction wave. If it reaches visible size (10^{-2}-10^{-1} cm) [9] we consider that cavitation has developed. Here the condition of the appearance of cavitation essentially depends on the correctness of the choice of $\pi^0 p_0$ (the maximum pressure in the rarefaction wave in absolute value) and T. The value of R_0 can be taken from the well-known experimental data of [8].

Within the framework of the acoustical model we can estimate the pressure near the free surface in the region of regular reflection during shallow underwater explosions ($H \lessgtr 10a_0$ is the depth of submersion of the charge of radius a_0). When the principle of superposition is used the negative pressure at a specific point of the region under consideration will be determined by the time which elapsed from the moment the shock-wave front (SWF) passed this point until the arrival of the rarefaction wave and by the nature of the pressure drop behind the shock-wave front. Consequently, for shallow explosions the lag time for the region of $r_1 \leq H$ should allow for the velocity of the shock-wave front up to the time of its reflection from the surface. Finally, the pressure in dimensionless form is determined by the expression

$$p = A(r_1) r_1^{-\alpha(r_1)} \begin{cases} e^{-\beta} & (\beta \leqslant 1) \\ 0.368\beta^{-1} & (\beta > 1) \end{cases} - A(r) r^{-\alpha(r)}, \tag{1.5}$$

where

$$\beta = \frac{\int_{r_1}^{H} U^{-1} dx + D(r-H)/c_0}{5.88 \cdot 10^{-6} D m r_1^{0.22}}, \quad r_1 \leqslant H;$$
$$\beta = \frac{r - r_1}{5.88 \cdot 10^{-6} c_0 m r_1^{0.22}}, \quad r_1 > H; \tag{1.6}$$

$m = (\frac{4}{3}\pi\rho_{EC})^{0.26}$; c_0 is the velocity of sound in the undisturbed liquid; D is the detonation velocity of the explosive charge (EC); U is the velocity of the shock-wave front with respect to D, all the linear values are normalized to a_0, and the subscript 1 pertains to the coordinate of a point in the system connected with the real charge. In Eq. (1.6) we use the expressions for

$$\theta_1^0(r) = 97.6 G^{1/3}(G^{1/3} r^{-1})^{-0.22};$$
$$G^{1/3} = 0.1 a_0 \sqrt[3]{\frac{4}{3}\pi\rho_{EC}},$$

where $\theta_1^0(r)$ is the decay constant behind the shock-wave front, μsec; G is the weight of the charge, kg; r is in m; ρ_{EC} is the density of the EC, g/cm^3; a_0 is in cm. The constants A and α are determined, for example, from the following data [6, 12]:

$$
\begin{array}{llllll}
r = & 1-1.13; & 1.13-2.31; & 2.31-4.0; & 4.0-12; & >12; \\
A, _{\text{atm}} = & 1.82\cdot 10^5; & 1.325\cdot 10^5; & 9\cdot 10^4; & 3.7\cdot 10^4; & 1.47\cdot 10^4; \\
\alpha = & 5.4; & 2.6; & 2.13; & 1.5; & 1.13.
\end{array}
$$

According to estimates which have been made, for the expansion of a cavitation bubble to visible size in a time interval T it is necessary that the amplitude of the negative wave be no lower than that determined by Eq. (1.3). We call this the critical pressure π_* and from it we estimate the upper boundary of the cavitation zone. Near the free surface we have $\pi_* \simeq -A\,(r)\,r^{-\alpha(r)}\beta$ and after simple transformations we obtain the following expression for the coordinates of the boundary:

$$
y \simeq \frac{1.78\pi_* \, (x^2 + H^2)^{0,61+\alpha/2}}{mc_0 HA}, \tag{1.7}
$$

where all the linear values are expressed with respect to a_0 and $A = A(\sqrt{x^2 + H^2})$. For the case of $a_0 = 0.53$ cm, $\rho_{EC} = 1.6$ g/cm^3, H = 3 cm, and $R_0 = 10^{-6}$ cm the critical pressure is $\pi_* = 100$ and x and y have the following values:

$$
\begin{array}{llllllll}
y, _{\text{mm}} = & 0.3; & 0.5; & 0.8; & 1.3; & 1.9; & 2.6; & 3.5; \\
x, _{\text{mm}} = & 16.9; & 28.2; & 39.5; & 50.8; & 62; & 73.3; & 84.6.
\end{array}
$$

The pressure region around the explosion cavity calculated from (1.5) with the conditions indicated above is presented in Fig. 1 and the position of the rarefaction-wave front (RWF) at different times is marked; $\delta(x)$ (dashed line) is the upper boundary of the cavitation zone according to (1.7); $\delta_1(x)$ (crossed line) is the upper boundary of the cavitation zone obtained through an approximation of certain solutions for the trajectory of the "triple" point in the case of irregular reflection of the shock wave from the free surface S.

In this formulation the negative pressures turn out to be unrealistically large, while the determination of the moment of appearance of a visible bubble and of the boundary of the cavitation zone requires the introduction of a number of additional assumptions which are considerably more difficult to justify.

2. Two-Phase Model of Development of Cavitation Zone

Let us consider the problem of the formation of the cavitation zone within the framework of the model of a two-phase medium consisting of a liquid containing cavitation nuclei, retaining the same principle of the superposition of the pressure field from an imaginary charge. While p (t) is known [6, 12] for a one-phase medium, in the case of a two-phase medium the pressure will depend on the volumetric concentration k (t) of free gas in the liquid and the relationship p (k (t)) has to be found. The pressure of a two-phase medium is described by a system of equations of hydrodynamics, a characteristic of which is the complicated form of the notation of the equation of state of the medium, including a nonlinear second-order equation for the pulsating cavitation bubble [11, 13-15].

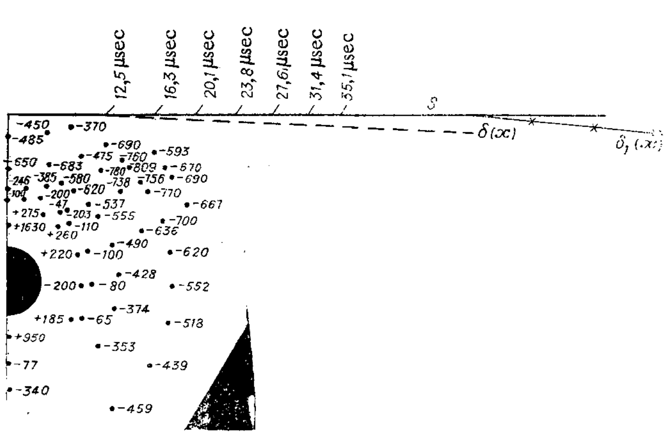

Fig. 1

We can write the system in the linearized form

$$\rho_t + u_x + v_y = 0, \quad u_t + p_x/3 = 0, \quad v_t + p_y/3 = 0,$$
$$\rho = (1 + k_0 k)^{-1}, \quad k_{tt} = -k^{1/3}(p - k^{-\gamma}), \tag{2.1}$$

where ρ, u, v, and p are the averaged values of the density, velocity components, and pressure in the medium; R_0 is the initial radius of the cavitation bubble; k is the volumetric concentration of gas in the medium; $u = \sqrt{\rho_0/3p_0}u'$;

$$v = \sqrt{\rho_0/3p_0}\,v'; \quad t = \sqrt{3p_0/\rho_0 R_0^2}\,t'; \quad x = x'/R_0;$$
$$y = y'/R_0; \quad k = k'/k_0; \quad p = p'/p_0; \quad \rho = \rho'/\rho_0;$$
$$k' = (R')^3.$$

The dimensional values are given primes. Here we assume that:

1) the characteristic dimension L of average motion, the average distance l between bubbles, and the bubble radius R satisfy the inequalities $L \gg l \gg R$;

2) the asphericity of the bubbles, the mass of the gas in them, and their motion relative to the liquid can be neglected;

3) the bubbles are the same size and are uniformly distributed in the liquid;

4) the liquid component of the medium is incompressible.

From (2.1) one can obtain

$$p_{xx} + p_{yy} + 3k_0 k_{tt} = 0;$$
$$k_{tt} = -k^{1/3}(p - k^{-\gamma}). \tag{2.2}$$

Let us introduce the new variables $x_0 = \sqrt{3k_0k^{1/3}}x$, $y_0 = \sqrt{3k_0k^{1/3}}y$ and the new unknown function $\zeta = p - k^{-\gamma}$. With allowance for the additional assumptions of the smallness of terms of the types $\gamma k^{-\gamma-1}k_{xx}$ and $xk_x/6$ in comparison with ζ_{xx} and k, respectively, from (2.2) we have

$$\Delta\zeta = \zeta. \tag{2.3}$$

An upper estimate of the error introduced by the additional assumptions can be made on the basis of the expression (1.3) for a single bubble.

In the case of the explosion of a spherical charge near a free surface the problem can be analyzed in an axisymmetric formulation, and then (2.3) in the polar coordinates r_0, θ has the form

$$r_0^{-2}\frac{\partial}{\partial r_0} r_0^2 \frac{\partial \zeta}{\partial r_0} + r_0^{-2}\sin^{-1}\theta \frac{\partial}{\partial \theta}\sin\theta\frac{\partial\zeta}{\partial\theta} = \zeta. \tag{2.4}$$

The solution is sought for in the form $\zeta = R^0\Theta$. From (2.4) we obtain the following equations for R^0 and Θ:

$$\frac{d}{dr_0}\left(r_0^2\frac{dR^0}{dr_0}\right) - \left[r_0^2 + \nu(\nu+1)\right]R^0 = 0;$$
$$\sin^{-1}\theta\frac{d}{d\theta}\sin\theta\frac{d\Theta}{d\theta} + \nu(\nu+1)\Theta = 0,$$

where the constant of separation of variables is denoted through $\nu(\nu + 1)$. The solution of these equations consists of spherical Legendre functions

$$\Theta = AP_\nu(\cos\theta) + BQ_\nu(\cos\theta)$$

and modified Bessel functions

$$R^0 = r_0^{-1/2}\left(CI_{\nu+1/2}(r_0) + DK_{\nu+1/2}(r_0)\right).$$

Because of the boundedness of the solution in the region under consideration, determined by the intervals of variation $0 \leq \theta \leq \pi$ and $r_0 > 0$, the coefficients B and C must be set equal to zero ($I(r_0) \to \infty$ as $r_0 \to \infty$, $Q(\cos\theta) \to \infty$ as $\cos\theta \to 1$). Finally, we can write the solution of (2.4) with $\nu = n$ ($n = 0, 1, 2, \ldots$) in the form

$$\zeta = r^{-1/2} \sum_{n=0}^{\infty} A_n K_{n+1/2}(r) P_n(\cos\theta). \tag{2.5}$$

Here and afterward the subscript zero is dropped from r.

With allowance for what has been said above, the problem of the development of the cavitation region is formulated as follows.

Suppose that in an unbounded liquid, which contains cavitation nuclei of radius R_0 with a volumetric gas concentration k_0, there are two cavities of radius a_0 containing the detonation products and located at the points O and O_1 at a distance h from one another. The two cavities can expand in accordance with an adiabatic law, the initial pressure in them is known and equal to p (0), and the values of $a(t)$ and the adiabatic index γ_1 of the detonation products are also known. We assume that $p(0) = p_{Im} = p^0_\alpha < 0$ at the point O and $p(0) = p_{Re} = p^0_{\alpha 1} > 0$ at the point O_1 and we phase shift the time of application of the pressure field from the explosion of the imaginary charge by the factor $\sigma_0(t - [r - r_1]/c_0)$, thereby modeling the delay in the arrival of the rarefaction wave at a given point. In such a case the pressure at any point of the medium is determined by the superposition of solutions of the type (2.5):

$$\zeta = r^{-1/2} \sigma_0 \sum_{n=0}^{\infty} A_n K_{n+1/2}(r) P_n(\cos\theta) + r_1^{-1/2} \sum_{n=0}^{\infty} B_n K_{n+1/2}(r_1) P_n(\cos\theta_1),$$

where $\sigma_0 = \begin{cases} 0 & t < (r - r_1)/c_0, \\ 1 & t \geqslant (r - r_1)/c_0; \end{cases}$ r and r_1 are the coordinates of the point under consideration in the systems with centers at O and O_1, respectively; c_0 is the dimensionless velocity of sound in the undisturbed liquid; the coefficients A_n and B_n are found from the conditions at the boundaries of the cavities containing the detonation products:

$$\zeta = p_\alpha(t) < 0, \ r = \alpha a(t),$$
$$\zeta = p_{\alpha 1}(t) > 0, r_1 = \alpha a_1(t).$$

Here $\alpha = \sqrt{3k_0 k^{1/3}}\, a_0/R_0$ while p_α and $p_{\alpha 1}$ are the pressures which are known at any moment. Dropping the awkward expressions for the coefficients A_n and B_n and assuming that $\alpha a_1 \ll h$, we finally find that

$$\zeta \simeq \frac{a}{r} p_\alpha \sigma_0 e^{-\alpha(r-a)} + \frac{a_1}{r_1} p_{\alpha 1} e^{-\alpha(r_1 - a_1)}.$$

Here and later all the linear values are expressed with respect to a_0: $r = r'/a_0$, $r_1 = r_1'/a_0$, $a = a'/a_0$. This expression in the first approximation determines the unknown relation p(k).

Thus, the problem of the development of cavitation near a free surface comes down to the solution of a system of equations relative to p and k:

$$(p - k^{-\gamma})/p^0_{\alpha 1} = \frac{a_1^{-3\gamma_1+1}}{r_1} e^{-\sqrt{3k_0 k^{1/3}}\frac{a_0}{R_0}(r_1 - a_1)} - \frac{a^{-3\gamma_1+1}\sigma_0}{\sqrt{r_1^2 + h^2 - 2hr_1\cos\theta_1}} e^{-\sqrt{3k_0 k^{1/3}}\frac{a_0}{R_0}\left(\sqrt{r_1^2 + h^2 - 2hr_1\cos\theta_1} - a\right)}; \tag{2.6}$$

$$\frac{d^2k}{dt^2} = -k^{1/3}(p - k^{-\gamma}) + \left(\frac{dk}{dt}\right)^2 \Big/ 6k,$$

where

$$t = 0 \ k = 1, \ \dot{k} = 0, \ a = 1,$$

and a_1 is determined by the following empirical functions for small explosive charges:

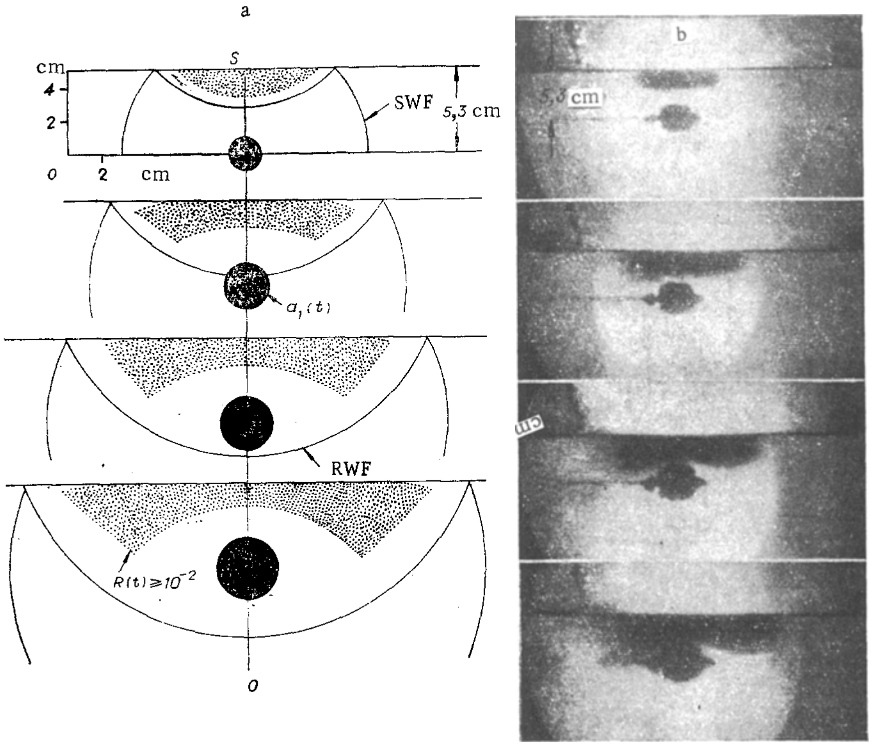

Fig. 2

$$a_1 \simeq 1 + 0.022 \cdot 10^6 \tau / a_0, \quad \tau < 10^{-4} \text{ sec},$$
$$a_1 \simeq 158.5 (\tau / a_0)^{0.4}, \quad \tau \geqslant 10^{-4} \text{ sec},$$

where $\tau = t + (r - r_1)/c_0$ (the latter expression for a_1 is taken from [16]).

As the calculation showed, the cavitation nuclei reach visible sizes (10^{-2}–10^{-1} cm) after a short time and therefore in many cases $\alpha(t)$ can be set equal to unity. It was mentioned earlier that [8] contains information, generalized on the basis of numerous experiments, on the state of free gas in liquids, which can be employed for the selection of the proper k_0 and R_0. For example, for water which has been standing $k_0 = 10^{-12}$–10^{-10} and $R_0 = 5 \cdot 10^{-5}$ cm, while for relatively fresh water $k_0 = 10^{-9}$–10^{-8} and $R_0 = 5 \cdot 10^{-3}$ cm.

Calculations of the development of the cavitation zone were performed for different R_0, k_0, a_0, and h. In Fig. 2a we present the results of the calculation of the visible cavitation zone (darkened region) for t = 16, 32, 48, and 64 μsec with $h'/2 = 5.3$ cm, $k_0 = 10^{-11}$, $R_0 = 5 \cdot 10^{-5}$ cm, $a_0 = 0.53$ cm (1 g EC), $p(0) = 4 \cdot 10^4$ atm, and $\gamma_1 = 3$. The sizes of the cavitation nuclei in it reached $\geq 10^{-2}$ cm by the indicated times. Frames of high-speed photography of the development of the cavitation zone upon the explosion of a 1-g charge at a depth of 5.3 cm are shown for comparison in Fig. 2b for the same times as in Fig. 2a. In the solution of system (2.6) the dynamics of the bubbles was taken into account only in the phase of negative pressure because of the extreme smallness of k_0 and R_0.

The system (2.6) also allows one to determine the profile of the rarefaction wave in the cavitation zone. As the calculation showed, $p(k(t))$ depends essentially on the rise time of the rarefaction-wave front, which is regulated by the factor σ_0 in the first equation of (2.6). The value of σ_0 can be taken as unity at the time t = 0 (the case of the instantaneous application of the maximum negative pressure) or represented in the form of a time function which determines the law of pressure rise in the rarefaction-wave front – the "pile-up" of the front. The latter is determined either experimentally or, for example, numerically on the basis of the data of [2, 6] from the difference in the times of arrival at the point under consideration of the characteristics of the rarefaction wave with zero and maximum amplitudes. In Fig. 3 we present the p(t) profiles for three relative distances from the center of the charge on the axis of symmetry calculated from (2.6) with $k_0 = 10^{-11}$, $R_0 = 5 \cdot 10^{-5}$ cm, $h'/2 = 3$ cm, and $p(0) = 4 \cdot 10^4$ atm. The rarefaction-wave profiles in the case of a one-phase liquid are shown by dashed lines, while those calculated from the two-phase model for a rise time of 0.1 μsec for the rarefaction-wave front are shown by solid lines. Profiles of the

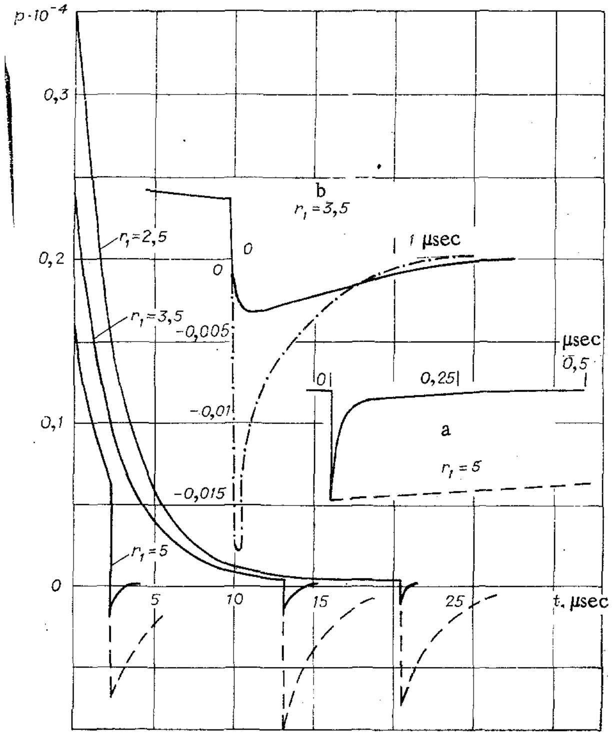

Fig. 3

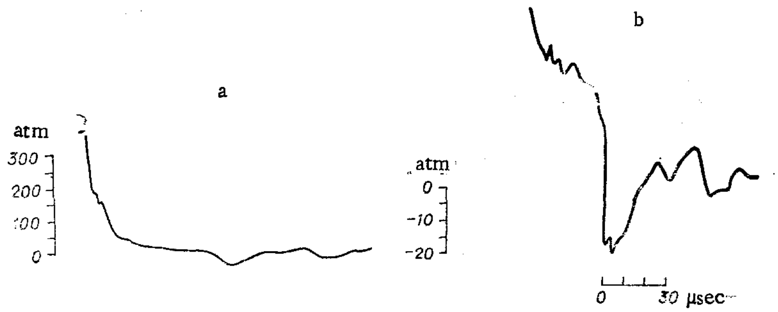

Fig. 4

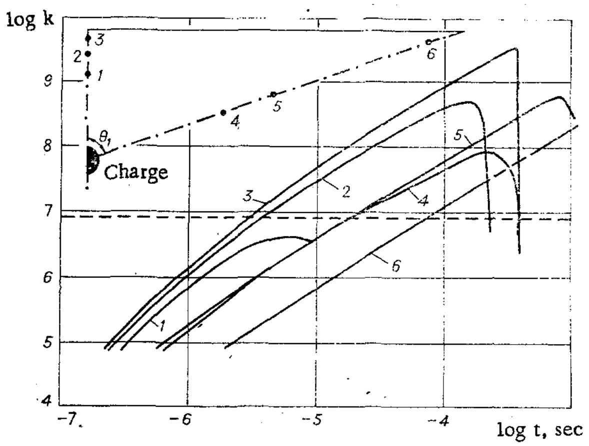

Fig. 5

rarefaction wave with different "pile-ups" of its front are shown in the middle of Fig. 3 for two cases: Fig. 3a with $r_1 = 5$, the negative pressure is applied instantaneously, the dashed line is the profile in a one-phase liquid, and the solid line is the profile in a two-phase liquid; it is seen that large negative pressures are retained in the cavitation zone for about 0.05–0.1 μsec (the calculation coincides with the known experimental data); Fig. 3b with $r_1 = 3.5$; 0.1-μsec "pileup" of the front, dashed-dot line (corresponds to the solid line in the main drawing for the same r_1), maximum amplitude has decreased more than fivefold in comparison with a one-phase liquid; 1-μsec "pileup" of front, solid line, here the maximum amplitude is already 30 times smaller than in a one-phase liquid.

In Fig. 3a the negative pressure has almost completely disappeared in a time of ~0.1 μsec, while the cavitation bubble has not even reached its visible size by this time; in Fig. 3b the pressure disappears when the bubble has expanded to about half of its maximum radius.

Oscillograms of the pressure taken at a distance of 14 cm from a 1-g charge in "deep" water (Fig. 4a) and at a distance of 4.5 cm below the free surface over the charge (Fig. 4b) are presented for a qualitative comparison in Fig. 4. The amplitude of the maximum pressure in the shock wave (Fig. 4a) is about 380 atm; the amplification scale for the measurement of pressure in the rarefaction wave (Fig. 4b) is increased by 10 times; the experimental value of the maximum amplitude of the rarefaction wave is 20 atm, while that calculated by the scheme of an imaginary source for a one-phase liquid is 217 atm.

In Fig. 5 we show the dependences k(t) with $a_0 = 0.53$ cm, $h'/2 = 5.3$ cm, $k_0 = 10^{-11}$, $R_0 = 5 \cdot 10^{-5}$ cm, and $p(0) = 4 \cdot 10^4$ atm for points 1-3 on the axis of symmetry and points 4-6 along a radial line at an angle $\theta_1 = 70°$ to the axis: for all the curves plotted the time t = 0 corresponds to the moment of arrival of the rarefaction wave front at the given point. The position of the charge and of points 1-6 relative to the free surface are shown in Fig. 5 for clearness. The gas concentrations corresponding to the visible size of a cavitation bubble are marked by a dashed line. The time interval between the two moments when the k(t) curve crosses the dashed line determines the "lifetime" of a visible bubble at the given point: the visible cavitation disappears in about 400 μsec at the axis and at point 4 on the radial line.

We note that at a given moment the gas concentration can vary by an order of magnitude or more over a cross section of the cavitation zone, which corresponds to different intensities of darkening of the zone.

The calculated results presented give reason to assume that the proposed two-phase model of the development of the cavitation zone satisfactorily describes the process of development of the zone and makes it possible to construct a rarefaction-wave profile which is close to the actual profile.

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